ASSIGNMENT 2

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1.

a) The probability of getting 1 when a dice is rolled is 1/6 =0.166

b) Probability of getting two 1’s in two different dice is 1/6 \* 1/6 = 1/36 = 0.027

2.

a) False. The probability of having a boy or a girl is always 50%, regardless of the gender of previous children. The gender of previous children does not affect the gender of future children.  
b) False. Drawing a face card and drawing a red card are not mutually exclusive events. There are red face cards in a deck of playing cards, so it is possible to draw a card that is both a face card and red.

c) True. Drawing a face card and drawing an ace are mutually exclusive events. A card cannot be both a face card and an ace.

3.

a) The male parent can only pass on his X chromosome, and the female parent can only pass on her X- chromosome. Therefore, the probability of their offspring being colorblind male is 1/2 \* 1/2 = 1/4.

b) False. Colorblindness and female sex are not mutually exclusive events. A female can be colorblind if she inherits two defective copies of the gene.

4.

a) The events are neither disjoint nor independent. The events are not disjoint because it is possible for both events to occur at the same time, meaning that both you and the randomly selected student can earn A's in the course. The events are not independent because the probability of the second event (the randomly selected student earning an A) depends on the outcome of the first event (you earning an A). Therefore, the events are dependent.

b) The occurrences are connected and could perhaps be independent. Because both events might happen simultaneously, they are not mutually exclusive, and you and your classmate could both receive A’s for the course. The degree to which your grades and those of your class study companion are connected determines whether the occurrences are independent or not. The events are independent if your grades are unrelated to one another, that is, if one person's grade has no bearing on the other person's grade. The events are dependant, however, if your grades are related, that is, if one person's grade influences the other person's grade.

c) No, two events that can occur at the same time are not necessarily dependent. Whether two events are dependent or independent depends on whether the occurrence of one event affects the probability of the occurrence of the other event.

5.

a) 25%, 15%, 28% of students miss exactly 1 day, 2 days, and 3+ days respectively. Probability that a student chosen at random doesn’t miss any days of school due to sickness is 100-25-15-28=32% which is 0.32.

b) probability that a student chosen at random misses no more than one day is student miss 0 days + student miss 1 day = 32%+25%= 57% which is 0.57.

c) The probability that a student chosen at random misses at least one day is 25%+15%+28%=68% which is 0.68.

d) probability that neither kid will miss any school is:

* probability that a student doesn’t miss any day = 0.32
* probability that neither kid miss any day is 0.32\*0.32 = 0.1024.

6.

a) Marginal probability that a randomly chosen individual always wears seatbelts is 375264/436968 = 0.85.

b) Probability that a randomly chosen female always wears seatbelts is 229246/255980 = 0.89.

c) Conditional probability of a randomly chosen individual always wearing seatbelts, given that they are female is 229246/255980 = 0.89.

d) Conditional probability of a randomly chosen individual always wearing seatbelts, given that they are male is 146018/180988 = 0.80.

e) probability that an individual who never wears seatbelts is male is 4719/7394 = 0.63.

f) They don’t seem to be dependent.

7.

The statement "It's never lupus" from the TV show House is not entirely true. While lupus is a rare disease, it is still possible for a patient to have it. The accuracy of the test for lupus can help us understand the probability of a positive test result being a true positive or a false positive. Let’s assume that 2% of the population has lupus. Using the information given in the question, we can calculate the following probabilities:

The probability of a positive test result given that the patient has lupus (true positive rate or sensitivity) is 98%.

The probability of a negative test result given that the patient does not have lupus (true negative rate or specificity) is 74%.

The probability of a positive test result given that the patient does not have lupus (false positive rate) is 26% (100% - 74%).

The probability of a negative test result given that the patient has lupus (false negative rate) is 2% (100% - 98%).

Using these probabilities, we can calculate the probability of a positive test result being a true positive or a false positive:

The probability of a true positive is the product of the prevalence of the disease (2%) and the sensitivity of the test (98%), which is 0.0196 or 1.96%.

The probability of a false positive is the product of the complement of the prevalence of the disease (98%) and the false positive rate of the test (26%), which is 0.2548 or 25.48%.

Therefore, if a patient tests positive for lupus, there is a 1.96% chance that they actually have the disease, and a 25.48% chance that the positive test result is a false positive. This means that while lupus is a rare disease, it is still possible for a patient to have it, and a positive test result should not be dismissed outright as "never lupus".

8.

a) To calculate positive predictive value (PPV) and negative predictive value (NPV), the following formulas can be used:

* PPV = (sensitivity x prevalence) / [ (sensitivity x prevalence) + ((1 – specificity) x (1 – prevalence))]
* NPV = (True Negatives) / (True Negatives + False Negatives)

Sensitivity = 0.2

Specificity = 0.94

i) For < 50 years,

prevalence = 0.001

PPV = 0.0329

True Negative = (1-0.94) \* (1-0.001) = 0.0594, False Negative = 0.2 \* 0.001 = 0.0002

NPV= 0.997

ii) 50-60 years

prevalence = 0.020

PPV = 0.45

NPV = 0.98

iii)

61-70 years

Prevalence = 0.060

PPV = 0.2207

NPV = Specificity×(1−Prevalence)/ ((1−Sensitivity) \* Prevalence) +(Specificity \* (1−Prevalence)).

NPV = (0.94 \* (1 - 0.060)) / (0.94 \* (1 - 0.060) + (1 - 0.2) \* 0.060) = 0.9707

NPV = 0.9707

iv) 71-80 years

Prevalence-0.100

PPV = (0.2 \* 0.100) / [(0.2 \* 0.100) + (1 - 0.94) \* (1 - 0.100)]

PPV = 0.169

NPV = (0.94 \* (1 - 0.100)) / [(1 - 0.2) \* 0.100 + (0.94 \* (1 - 0.100))]

NPV = 0.971

b) As the prevalence of the disease rises, there is a corresponding increase in PPV values. Conversely, as the prevalence of the disease decreases, NPV values also tend to rise.

c) As a man gets older, he is more likely to have undetected prostate cancer. So, a positive PSA test is more likely to correctly detect cancer (higher PPV) in older men compared to younger men. On the other hand, a negative PSA test is slightly less reassuring (lower NPV) in older men compared to younger men, because the cancer may be missed due to the imperfect sensitivity of the test.

d) Lowering the cutoff for a positive PSA test would increase the sensitivity of the test, meaning more men with prostate cancer would test positive. However, it would also decrease the specificity, meaning more men without prostate cancer would also test positive. So there would be more true positives but also more false positives if the cutoff was lowered.

9.

(a) Since both parents have brown eyes, they must both have at least one dominant B allele. There are three possible parental genotype combinations: BB x BB BB x Bb Bb x Bb

In the BB x BB case, the probability of a bb child is 0%. In the BB x Bb case, the probability of a bb child is 50%. In the Bb x Bb case, the probability of a bb child is 25%.

Since we don't know the parental genotypes, by calculating the probability as a weighted average: 11%

Therefore, the probability of their first child having blue eyes is 0.11.

(b) Knowing that the paternal grandfather has blue eyes tells us that the father must have at least one recessive b allele. This eliminates the possibility that the father is BB homozygous dominant. Therefore, the only possible parental genotype combinations are: Bb x Bb Bb x bb

In both cases, the probability of a bb child is 2/3(25%). So the probability does not change and is still 0.16.

(c) Given their first child has brown eyes, either both parents are Bb or one is Bb and one is BB. In either case, there is a 25%-11% probability of having a bb child. So the probability their second child has blue eyes is 0.25-0.11 ~ 0.12.

10.

a) No, being in excellent health and having health coverage are not mutually exclusive.

b) probability that a randomly chosen individual has excellent health is 0.2329/1 = 0.2329.

c) the probability that a randomly chosen individual has excellent health given that he has health coverage is 0.2099/0.8738 = 0.24.

d) the probability that a randomly chosen individual has excellent health given that he doesn’t have health coverage is 0.0230/0.1262 = 0.18.

e) No, having excellent health and having health coverage do not appear to be independent.